Estimation of Aerodynamic Forces and Moments on a Steadily Spinning Airplane

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A semiempirical method is presented for the estimation of aerodynamic forces and moments on a steadily rotating airplane model in a spin tunnel. The approach is based on the application of strip theory to determine a part of the aerodynamic coefficient (including rotational velocity) and then estimation of increments to these coefficients because of rotational flow over the stalled airplane. The theory is applied to a light, single-engine, general aviation airplane and the results are compared with the corresponding spin tunnel rotary balance test data.

Nomenclature

A_s	= shielded area
b	= wing span
b_h	= local width of fuselage cross section
b_t	= horizontal tail span
c	= wing chord
c_t	= horizontal tail chord
C_A	= axial force coefficient = axial force (A)/ $\frac{1}{2}\rho V^2 S_w$
c_f	= flat plate chord
C_ℓ	= rolling moment coefficient = rolling moment/ $\frac{1}{2}\rho V^2 S_w b$
C_L	- lift coefficient = lift $(L)/\frac{1}{2}\rho V^2 S_w$
C_n	= yawing moment coefficient = yawing moment/ $\frac{1}{2}\rho V^2 S_w b$
$1C_N$	= normal force coefficient = normal force $(N)/\frac{1}{2}\rho V^2 S_w$
c_t	= horizontal tail chord
c_v	= vertical tail chord
C_{xc}	= axial force coefficient of noncircular cross section
C_Y	= side force coefficient = side force $(Y)/\frac{1}{2}\rho V^2 S_w$
d_f	= maximum width of near wake
h_v	= vertical distance between vertical tail center of pressure and center of gravity
k	= apparent mass coefficient of the body
ℓ	= length of the fuselage
ℓ_t	= horizontal tail length
ℓ_v	= vertical tail length
p	= pressure
p_c	= pressure in the wake
q	= freestream dynamic pressure = $\frac{1}{2}\rho V^2$
r_v	= streamwise coordinate of a stagnation streamline
S	= area
$S_{B\max}$	= maximum cross-sectional area of fuselage or body
V	= freestream velocity
vol	= volume of fuselage
x_{cg}	= airplane center of gravity location, measured from leading edge of fuselage
$\bar{x}_{cp}$	= center of pressure of wing in terms of chord
α	= angle of attack
β	= sideslip angle
Δ	= strip parameter
ϵ	= correction factor for three-dimensional effects over body
θ	= $\tan^{-1}(\Omega y/V)$
ϕ	= cross flow angle = $\tan^{-1}(\tan\beta/\sin\alpha)$
η_t	= tail efficiency = q_t/q
η_s	= $2y_s/b$, the spanwise extent of stall
λ	= $\tan^{-1}\omega$
ρ	= density
ω	= reduced spin rate = $\Omega b/2V$
Ω	= angular velocity about spin axis

Superscripts

( )	= parameter in rotational flow effect

Subscripts

B	= body or fuselage
ℓ	= strip
L	= left
R	= right
stall	= condition or parameter at stall
t	= tail
v	= vertical tail
w	= wing

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Introduction

THE prediction and analysis of airplane stall/spin characteristics has been of great interest to designers since the beginning of aviation. This problem has assumed more importance in recent years of significant losses that have occurred to military and general aviation aircrafts because of out-of-control motions associated with stall/spin.[1] Although considerable progress has been made in recent years in the areas of experimental and flight testing techniques related to stall/spin technology, a major obstacle remains because of the lack of an adequate mathematical model for simulating aerodynamic forces and moments that occur in spin maneuvers.

Early investigators[2-4] made attempts to estimate the aerodynamic characteristics of a spinning airplane. Since such characteristics are quite complex, they employed a semiempirical approach based on strip theory.[5] Their efforts were limited to the analysis of spinning motions occurring at combined low angles of attack and spin rates (steel spin) where the autorotation of stalled wings is the driving mechanism in spin. The problem of flat spin, where both the angle of attack and spin rates are high and yawing moment characteristics of fuselage assume importance, was not given much attention Because the early investigators were interested mainly in the steep spin problem, no attention was given by them to the task of predicting fuselage aerodynamic characteristics Nonavailability of an adequate aerodynamic mathematical model led to the development of the spin tunnel rotary balance apparatus for generating the pertinent aerodynamic test data [6] It has now been demonstrated that by incorporating the spin tunnel rotary balance test data into six degree of freedom equations of motion the steady state spins can be successfully predicted.[7,8] Thus we now have reliable experimental data generated by rotary balance apparatus that can serve as a guiding factor in the development of an aerodynamic mathematical model for steady state spins, which forms the subject matter of this investigation

In the present study, an effort has been made to estimate the aerodynamic forces and moments acting on an airplane model steadily rotating in a vertical spin tunnel, analogous to a model undergoing spin tunnel rotary balance test Extreme angles of attack (up to 90 deg) and reduced spin rates (0 to 1) are covered in the analysis The airplane model is considered to be divided into wing fuselage (body) and horizontal and vertical tail surfaces The effect of power is ignored The estimation of each of the aerodynamic forces and moments for a given airplane component consists of two steps: 1) strip theory calculations and 2) increments to these coefficients on account of rotational flow effects The net or total aerodynamic coefficient is considered to be the algebraic sum of these two quantities

The strip theory approach for wing and horizontal tail surfaces is similar to that used in earlier studies.[2,3] For the fuselage a semiempirical approach is developed for the prediction of static aerodynamic characteristics at combined high angle of attack and side slip This procedure is then extended to the case of a steadily spinning fuselage based on strip theory concept similar to that suggested by Munk.[9] A mathematical model of the shielding effect over the vertical tail surface is developed based on experimental data for the prediction of vertical tail aerodynamic characteristics at high angle of attack Increments to above coefficients because of rotational flow over wing and tail surfaces are evaluated from fluid dynamic considerations The rotational flow effects are ignored for the body The total aerodynamic coefficient of the airplane is obtained by summing up the contribution from all components of the airplane The analysis is quite general; in this paper however, some specific applications are discussed

The theory is applied to a single engine light general aviation airplane, called "model A" airplane which has been extensively studied[10,14] in the stall/spin program of NASA Langley Research Center In this paper the results are presented for two different tail configurations (Tail Nos 3 and 4) at those angles of attack around which steady state spin modes[10] are found to exist These results are compared with the corresponding spin tunnel rotary balance test data [12,13] In these tests,[12] the spin radius is set to zero for $30<\alpha$ deg <90 and the reduced spin rate varies from 0 to ± 0.9 It is shown that the present theory gives good prediction of aerodynamic characteristics for steep and moderately steep spin modes but needs further improvement for application to flat spin problems

A NASA publication,[30] giving complete details of the calculations and results for various component configurations such as body wing body horizontal tail, etc , similar to those found in rotary balance tests[12] is under preparation

Analysis

Let us consider an airplane model, mounted on a rotary balance apparatus in the spin tunnel and steadily rotating at an angular velocity (Ω) as shown in Fig 1 Let us assume that the y axis of the airplane model lies in the horizontal plane and the spin radius is zero We also assume that the power is off and the control (elevator, rudder, and aileron) deflections are zero

Wing Contribution

Strip Theory Calculations

The effect of the angular velocity in spin (Ω) will be to induce at any spanwise location (y) a chordwise velocity component equal to Ωy (Fig 1) Let dy be the spanwise width of the strip Let us assume that the airplane model is in a right spin ($\Omega>0$). Due to rotation, the local angle of attack (α_ℓ) on the right wing will increase ($\alpha+\theta$) and on the left wing will decrease ($\alpha-\theta$) compared to the angle of attack at the root chord (α) The local dynamic pressure is given by

$$q_\ell = \tfrac{1}{2}\rho V^2\left[1+(\Omega y/V)^2\right] \tag{1}$$

The lift drag, normal, and chordwise forces on any strip are

$$\Delta L = \tfrac{1}{2}\rho V^2\left[1+(\Omega y/V)^2\right]C_{L\ell}c\,dy \tag{2}$$

$$\Delta D = \tfrac{1}{2}\rho V^2\left[1+(\Omega y/V)^2\right]C_{L\ell}c\,dy \tag{3}$$

$$\Delta N = \Delta L\cos(\alpha\pm\theta) + \Delta D\sin(\alpha\pm\theta) \tag{4}$$

$$\Delta A = -\Delta L\sin(\alpha\pm\theta) + \Delta D\cos(\alpha\pm\theta) \tag{5}$$

The aerodynamic coefficients of the wing can then be

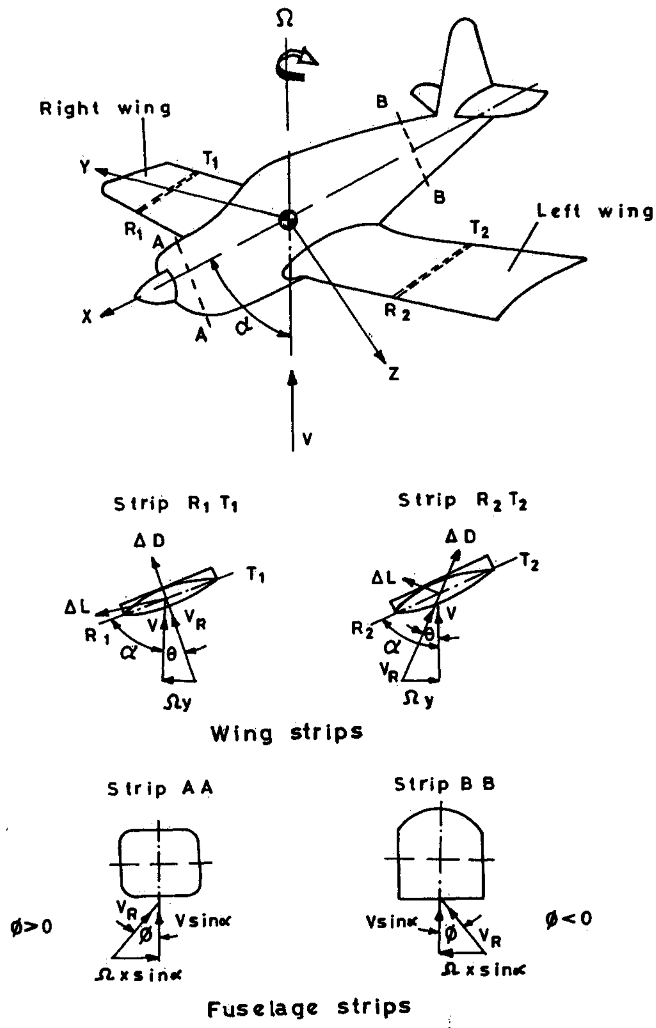


Fig 1 Schematic diagram of a steadily rotating airplane model in spin tunnel

derived as follows:

$$C_{Nw} = \frac{1}{2\tan\lambda}\int_o^\lambda (\Delta C_{NR} + \Delta C_{NL})\sec^4\theta d\theta \quad (6)$$

$$C_{Aw} = \frac{1}{2\tan\lambda}\int_o^\lambda (\Delta C_{AR} + \Delta C_{AL})\sec^4\theta d\theta \quad (7)$$

$$C_{\ell w} = \frac{1}{4\tan^2\lambda}\int_o^\lambda (\Delta C_{NL} \ \Delta C_{NR})\tan\theta\sec^4\theta d\theta \quad (8)$$

$$C_{nw} = \frac{1}{4\tan^2\lambda}\int_o^\lambda (\Delta C_{AR} \ \Delta C_{AL})\tan\theta\sec^4\theta d\theta \quad (9)$$

and

$$C_{mw} = \frac{1}{2\tan\lambda}\int_o^\lambda (\Delta C_{NR} + \Delta C_{NL})(\bar{x}_{cg} - \bar{x}_{cp})\sec^4\theta d\theta \quad (10)$$

The side force coefficient of the wings C_{yw}, is generally small and hence can be neglected In adition, we ignore the dihedral effect In the above equations C_L C_D and $\bar{x}_{cp}$ are two dimensional wing section characteristics, which may be determined from the data given in Refs 15 17 However, for the specific example considered here, the evaluation of these parameters is discussed later The chordwise variation of these coefficients is ignored

Rotational Flow Effects

Let us consider a stationary wing operating at $\alpha > \alpha_{\text{stall}}$ On the upper side the flow is completely separated (p p_c), and on the lower side the flow is normally attached If this wing is set in angular rotation, the fluid particles will experience a centrifugal force On the upper surface, the stagnant (stalled) fluid may be assumed to rotate (Fig 2) with the wing like a solid body in order to set up a spanwise pressure gradient to balance the centrifugal force On the lower side the centrifugal force may be balanced by a component of viscous stress and no spanwise pressure variation may be necessary The establishment of this spanwise pressure gradient (additional pressure differential (p pc) between upper and lower surfaces) would result in an increase in normal force and pitching moment. If the spanwise pressure differentials are unsymmetric, an increment to rolling moment will also come into existence These incremental quantities are referred to here as rotational flow effect For such a rotational flow model, McCormick[18] has presented the following expressions For both wings under fully stalled conditions he gives

$$C'_{Nw} = \frac{2\omega^2}{3} \qquad C_{\ell w} = 0 \quad (11)$$

For certain values of α and ω the left wing in right spin (vice versa in left spin) operates under partial stall (Fig 2) For such case McCormick[18] gives

$$C'_{Nw} = \frac{\omega^2}{3}(1 + \eta_s^3) \qquad \eta_s = 2y_s/b \quad (12)$$

$$C'_{\ell w} = \pm\frac{\omega^2}{16}(1 - \eta_s^2)^2 \qquad \Omega \gtrless 0 \quad (13)$$

An examination of McCormick's[18] equations revealed that Eq (13) is in error To derive the correct expression, we proceed as follows

For a right spin, the pressure distribution over the left wing operating under partial stall is given by

$$p = p_c + \tfrac{1}{2}\rho V^2\omega^2(\eta^2 - \eta_s^2) \quad (14)$$

such that at $\eta = -\eta_s$ $p = p_c$ as assumed by McCormick [18]

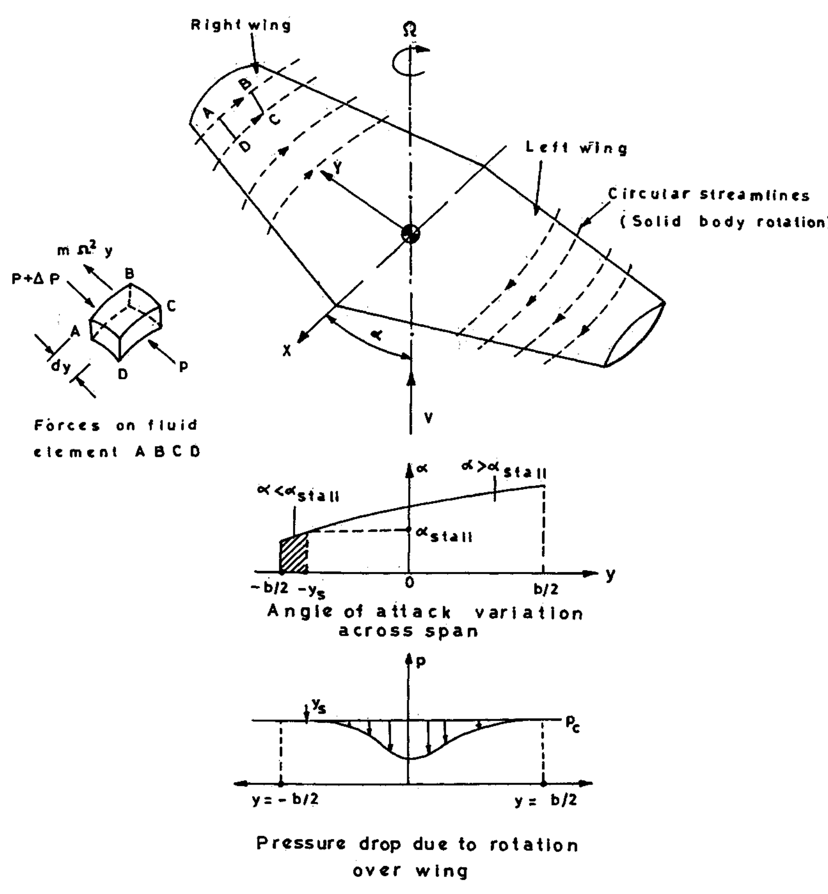


Fig 2 Rotational flow over stalled wing

For the fully stalled right wing we have

$$p = p_c + \tfrac{1}{2}\rho V^2 \omega^2 (\eta^2 - l) \tag{15}$$

so that at $\eta = 1$ $p = p_c$ It may be noted that Eqs. (14) and (15) check with corresponding McCormick's[18] equations Using the above pressure variations for left and right wings the correct expression can be derived as follows:

$$C'_{Nw} = \pm \frac{\omega^2}{16}(1 - \eta_s{}^4) \qquad \Omega \gtrless 0 \tag{16}$$

also,

$$C_{mw} = C'_{Nw}(\bar{x}_{cg}\bar{x}_{cp}) \tag{17}$$

Here, $\bar{x}_{cp}$ is the center of pressure of the force due to rotational flow effect and is asumed equal to 0 5 In this analysis the wing body interference is ignored

Fuselage Contribution

Past investigations[19-21] have shown that the cross sectional shape and Reynolds number have significant influence on the spin and recovery characteristics of the airplane's fuselages Cross sections having a rectangular or square shape with sharp corners generally produced autorotative (prospin) yawing moments in spin Rounding off the bottom corners results in a profound antispin effect, whereas rounding off only top corners does not alter the basic prospin tendency

In spin fuselage operates at a high angle of attack and also experiences a varying side slip At present, no analytical method is available for the prediction of aerodynamic characteristics of fuselage operating at combined high angle of attack and side slip Therefore, in the following we start with semiempirical procedures for a fuselage at 1) angle of attack, 2) side slip, and 3) combined angle of attack and side slip Then we extend this procedure to the case of a spinning fuselage For the analysis of static fuselage, the origin of coordinate system is chosen at the leading edge of the fuselage and x is regarded as positive when measured from the leading edge (Fig 3)

Fuselage at Angle of Attack

According to Allen,[22] for a body of revolution at a small angle of attack

$$C_{LB} = \frac{k\sin 2\alpha\cos\alpha/2}{S_{B\max}} \int_o^\ell \frac{dS_B}{dx}\,dx + \frac{2\sin^2\alpha\cos\alpha}{S_{B\max}} \int_o^\ell \epsilon r C_d\,dx \tag{18}$$

$$C_{DB} = \frac{k\sin 2\alpha\sin\alpha/2}{S_{B\max}} \int_o^\ell \frac{dS_B}{dx}\,dx + \frac{2\sin^3\alpha}{S_{B\max}} \int_o^\ell \epsilon r C_d\,dx \tag{19}$$

and

$$C_{MB} = \frac{k\sin 2\alpha\cos\alpha/2}{\mathrm{vol}} \int_o^\ell \frac{dS_B}{dx}({}_{cg} - x)\,dx + \frac{2\sin^2\alpha}{\mathrm{vol}} \int_o^\ell \epsilon r C_d (x_{cg} - x)\,dx \tag{20}$$

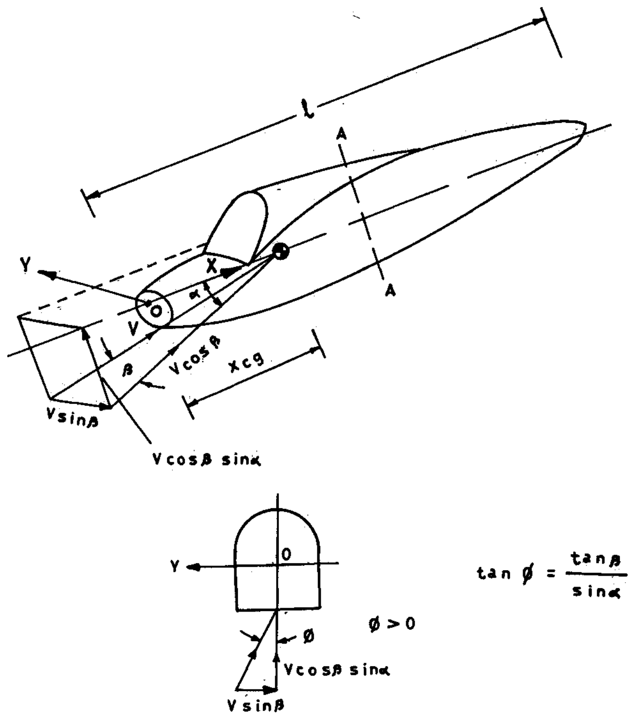


Fig 3 Static fuselage at combined α and β

In the above equations, k denotes the apparent mass coef ficient ($k = k_2 - k_1$ in Allen's[22] notation) and the zero lift drag appearing in Allen's[22] equation is ignored Jorgensen[23] proposes that Allen's[22] approach be adapted to noncircular bodies with the following modification:

$$C_{LB} = \frac{C_l k\sin 2\alpha\cos\alpha/2}{S_{B\max}} \int_o^\ell \frac{dS_B}{dx}\,dx + \frac{\sin^2\alpha\cos\alpha}{S_{B\max}} \int_o^l \epsilon b_h C_{xc}\,dx \tag{21}$$

$$C_{DB} = \frac{C_l k\sin 2\alpha\cos\alpha/2}{S_{B\max}} \int_o^l \frac{dS_B}{dx}\,dx + \frac{\sin^3\alpha}{S_{B\max}} \int_o^\ell \epsilon b_h C_{xc}\,dx \tag{22}$$

$$C_{mB} = \frac{C_l k\sin 2\alpha\cos\alpha/2}{\mathrm{vol}} \int_o^l \frac{dS_B}{dx}(x_{cg} - x)\,dx + \frac{\sin^2\alpha}{\mathrm{vol}} \int_o^l \epsilon b_h C_{xc}(x_{cg} - x)\,dx \tag{23}$$

The modifications are: 1) the first term is multiplied by a constant C_l so that the quantity $C_l k$ represents the actual apparent mass coefficient of a noncircular cross section; and 2) the coefficient C_{xc} based on b_h is the two dimensional cross flow drag coefficient of the noncircular cross sectional body It may be observed that for circular cross section, $C_l = 1$ 0, $b_h = 2r$ and $C_{xc} = C_d$ Jorgensen[23] also proposes that Eqs (21)-(23) be applied for $\alpha = 0$ 90 deg In the present analysis, all the concepts proposed by Jorgensen[23] are used

Fuselage in Side Slip

For a noncircular fuselage in side slip, we propose to extend

Jorgensen's[23] concept as follows:

$$C_{YB} = -\frac{C_1 k \sin 2\beta \cos\beta/2}{S_{B\max}} \int_o^l \frac{dS_B}{dx} dx + \frac{\sin^2\beta}{S_{B\max}} \int_o^l \epsilon b_h C_{yc} dx \tag{24}$$

$$C_{nB} = -\frac{C_1 k \sin 2\beta \cos\beta/2}{\text{vol}} \int_o^l \frac{dS_B}{dx} (x_{cg} - x) dx + \frac{\sin^2\beta}{\text{vol}} \int_o^\ell b_h \epsilon C_{yc} (x_{cg} - x) dx \tag{25}$$

In Eqs (24) and (25) all the terms are the same as before except that the first term has a negative sign and the side force coefficient C_{yc} is introduced in place of C_{xc} and is supposed to embody the side force characteristics of the noncircular two dimensional section Therefore, capability of this approach depends upon the availability of experimental data on C_{yc} for the given shape at proper Reynolds numbers

Fuselage at Combined Angle of Attack and Side Slip

For any given values of α and β we can define a cross flow angle ϕ such that

$$\tan\phi = \frac{\tan\beta}{\sin\alpha} \qquad \alpha \neq 0 \tag{26}$$

The above equation relates the two dimensional cross flow angle ϕ with three dimensional angle of attack (α) and side slip (β) (Fig 3) To evaluate aerodynamic coefficient under combined α and β first determine ϕ from Eq. (26) and then corresponding values of C_{xc} and C_{yc} for the subject cross section from the empirical data, supposed to be known Then integrate the expressions in Eqs (21) (25) to obtain the desired aerodynamic coefficients

Strip Theory Calculations for a Spinning Fuselage

The variation of cross flow angle and dynamic pressure along the length of a rotating fuselage are given by

$$\phi(x) = \tan^{-1}\left(\frac{\Omega x}{V}\right) \tag{27}$$

$$q_{iB}(x) = \tfrac{1}{2}\rho V^2 \left[1 + \left(\frac{\Omega x \sin\alpha}{b}\right)^2\right] \tag{28}$$

To evaluate the aerodynamic coefficients of a spinning fuselage, we divide it into a convenient number of axial strips (20 or 25) Then determine ϕ and q_{iB} from Eqs (27) and (28) Next we assume that the entire fuselage operating at a given α, experiences these values of ϕ and corresponding β[Eq (26)] and determine the strip aerodynamic coefficients from the following equations

$$\Delta C_{NB} = \frac{C_1 k \sin 2\alpha \cos\alpha/2}{S_{B\max}} \left(\frac{dS_B}{dx}\right) + \frac{\sin^2\alpha}{S_{B\max}} \epsilon b_h C_{xc} \tag{29}$$

$$\Delta C_{mB} = \Delta C_{NB} (x_{cg} - x) \left(\frac{S_{B\max}}{\text{vol}}\right) \tag{30}$$

$$\Delta C_{YB} = -\frac{C_1 k \sin 2\beta \cos\beta/2}{S_{B\max}} \left(\frac{dS_B}{dx}\right) + \frac{\sin^2\beta}{S_{B\max}} \epsilon b_h C_{yc} \tag{31}$$

$$\Delta C_{nB} = \Delta C_{YB} (x_{cg} - x) \left(\frac{S_{B\max}}{\text{vol}}\right) \tag{32}$$

Repeat this procedure for all the strips and numerically in tegrate the following equation

$$C_{iB} = \int_o^\ell \left[1 + \left(\frac{\Omega x \sin\alpha}{V}\right)^2\right] \Delta C_{iB}(x)\, d(x/\ell) \tag{33}$$

Where $i = 1$ 5 represents each of the aerodynamic coef ficients except the rolling moment coefficient ($C_{\ell B}$) which is assumed equal to zero The above strip theory procedure for fuselage in spin is similar to that suggested by Munk [9]

Clarkson et al [24] have found that the effect of rotation is negligible on the aerodynamic characteristics of certain noncircular fuselages Although in their tests, reduced spin rates varied approximately from -0 2 to 0 2 for the lack of more comprehensive information, we ignore the rotational flow effects over fuselage

Horizontal Tail Contribution

For $\alpha > \alpha_{\text{stall}}$, as stated in Datcom,[15] one can assume that the flow from the wing does not affect the horizontal tail surface if the latter is not immersed in the wing wake ($\eta_t = 1$) The wake of a stalled wing can be approximately taken to be bounded by the lines emanating from the leading and trailing edges of the surface in the streamwise direction When the horizontal tail surface is immersed in wing wake, its con tribution can be taken to be zero ($\eta_t = 0$) With these assumptions and proceeding similar to the analysis of the wing, we obtain

$$C_{Nt} - \frac{\eta_t S_t}{2S_w \tan\lambda_t} \int_o^\ell (\Delta C_{NRt} + \Delta C_{NLt}) \sec^4\theta_t d\theta_t \tag{34}$$

$$C_{At} = \frac{\eta_t S_t}{2S_w \tan\lambda_t} \int_o^\ell (\Delta C_{ARt} + \Delta C_{NLt}) \sec^4\theta_t d\theta_t \tag{35}$$

$$C_{mt} = -C_{Nt}(\ell_t / c) \tag{36}$$

$$C_{\ell t} = \frac{\eta_t}{4\tan^2\lambda_t} \left(\frac{S_t}{S_w} \frac{b_t}{b}\right) \int_o^{\lambda_t} (\Delta C_{NLt} - \Delta C_{NRt}) \times \tan\theta_t \sec^4\theta_t d\theta_t \tag{37}$$

$$C_{nt} = \frac{\eta_t}{4\tan^2\lambda_t} \left(\frac{S_t}{S_w} \frac{b_t}{b}\right) \int_o^{\lambda_t} (\Delta C_{ARt} - \Delta C_{ALT}) \times \tan\theta_t \sec^4\theta_t d\theta_t \tag{38}$$

$$C'_{Nt} = \frac{\omega^2}{3} \frac{S_t}{S_w} \eta_t (1 + \eta_{St}^3) \qquad \eta_{S_t} = \frac{2y_{st}}{b_t} \tag{39}$$

$$C'_{\ell t} = \pm \frac{\omega^2}{16} \eta_t \left(\frac{S_t}{S_w} \frac{b_t}{b}\right) (1 - \eta_{S_t}^4) \qquad \Omega \gtrless 0 \tag{40}$$

$$C'_{mt} = -C'_{Nt}(\ell_t / c) \tag{41}$$

Vertical Tail Contribution

In a steady state spin the aerodynamic flowfield at the vertical tail is affected by wing body side wash; loss in dynamic pressure due to drag of forward surfaces; mutual interference of aft body and tail surfaces; and mutual interference between tail surfaces (shielding) The effect of wing body side wash at high angles of attack can be ignored[4] and loss in dynamic pressure can be accounted[15] in the usual fashion through the tail efficiency (η_v) Very little information is available in literature on the subject of aerodynamic interferences at high angles of attack which are quite complex and not yet understood Here, we shall attempt to develop a mathematical model of the shielding effect over vertical tail and ignore the mutual interference between aft body and tail

Shielding Effect

On a spinning airplane the wake of the outboard panel of horizontal tail is pushed toward the vertical tail (Fig 4a), which results in the shielding of the vertical tail portion coming in its proximity The wake of the inboard panel is swept away from the vertical tail surface Thus, only the outboard panel could account for the shielding phenomenon Such an observation has also been made by Bihrle and Bowman [20] We assume that the characteristics of the wake of horizontal stabilizer are identical to that of a two dimensional flat plate and the shielding of vertical tail is caused by the "near wake " In the following, a semiempirical model of the near wake of a two dimensional flat plate is developed

Modeling of Near Wake

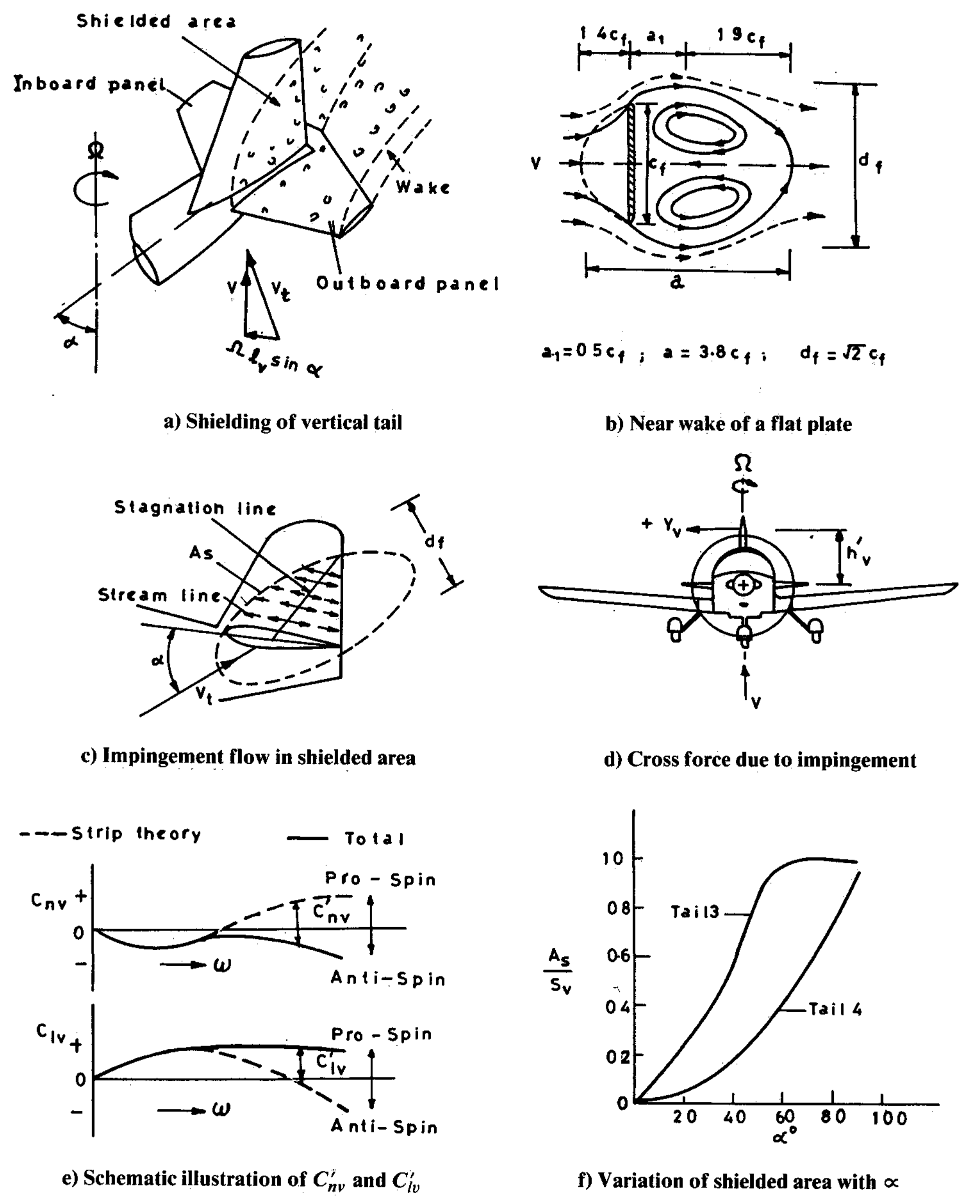
Arie and Rouse[25] show that the near wake of a normal flat plate (Fig 4b) consists of a closed bubble with an approximate elliptical shape, and extends to a downstream distance of about 2 4 times the flat plate chord Abernathy[26] gives the following relations for the other dimensions of the elliptical bubble (Fig 4b)

$$\frac{d_f(\alpha)}{d_f(\alpha=90\mathrm{deg}} = \sin\alpha(40\mathrm{deg}<\alpha< \quad \mathrm{deg})$$

$$d_f(\alpha=90\mathrm{deg})=\sqrt{2}\,c_f \tag{42}$$

as

$$a_l(\alpha=90\,\mathrm{deg})=0\ 5\,c_f \tag{43}$$



a) Shielding of vertical tail

b) Near wake of a flat plate

c) Impingement flow in shielded area

d) Cross force due to impingement

e) Schematic illustration of C'_{nv} and C'_{lv}

f) Variation of shielded area with α

Fig 4 Vertical tail in right spin

Combining these two results,[25 26] the elliptical shaped bubble for a normal flat plate has been sketched as shown in Fig 4b For application to other angles of attack, we assume that the correlation given in Eq. (42) holds for other dimensions also; that is,

$$a_I(\alpha) = 0\ 5c_f\sin\alpha \quad \text{and} \quad a(\alpha) = 3\ 8c_f\sin\alpha \tag{44}$$

Using the above empirical relationships [Eqs (42) (44)] the shielded area (A_s) can be determined However, for $\alpha_{\text{stall}} < \alpha < 40$ deg, linear extrapolation is assumed

Aerodynamic Coefficients

We make a direct approach instead of a strip theory because the vertical tail normally has a small aspect ratio

$$\beta_v = -\sin^{-1}\left\{\frac{2\beta(\ell_v/b)\sin\alpha}{\sqrt{1+[2\omega(\ell_v/b)\sin\alpha]^2}}\right. \tag{45}$$

and

$$q_v = \tfrac{1}{2}\rho V^2 \eta_v \left[1 + \left(\frac{2\omega\ell_v\sin\alpha}{b}\right)^2\right] \tag{46}$$

So that the side force coefficient, equivalent of strip theory is given by

$$C_{Yv} = \left[1 + \left(\frac{2\omega_{\ell v}}{b}\sin\alpha\right)^2\right]\eta_v C_{Yv}\left(\frac{S_v - A_s}{S_w}\right) \tag{47}$$

where C_{Yv} is the side force coefficient of unshielded vertical tail and its determination is discussed later

$$C_{\ell v} = C_{yv}\left(\frac{h_v}{b}\right) \tag{48}$$

$$C_{nv} = -C_{yv}\left(\frac{\ell v}{b}\right) \tag{49}$$

We assume

$$C_{Nv} = C_{mv} = C_{Av} = 0 \tag{50}$$

Rotational Flow Effect

On a spinning airplane, the mass of fluid in the wake of a horizontal stabilizer will have a relative transverse velocity equal to $\Omega l_v \sin\alpha$ with respect to the vertical tail and can be assumed to impinge perpendicularly on the windward side of the vertical tail forming a stagnation line about which the airstream will divide and move in opposite directions (Fig 4c) The existence of such a flow pattern has been noticed in the full scale spin flight tests conducted by NASA Langley Research Center

As a consequence of this wake flow impingement, which is limited to shielded area and is designated as rotational flow effect, a side force Y_v (Fig 4d) will develop on the fin because of pressure difference $(p_s - p_c)$ and generate an autorotative rolling moment and an antispin yawing moment (Fig 4e)

We assume that the above wake flow impingement is similar to that of the impingement of an axially symmetric flow of finite extent (e g , uniform jet) over a normal flat plate From Ref 27, close to stagnation point

$$p_s = p + \tfrac{1}{2}\rho U_o^2[1 - (r/r_j)^2] \tag{51}$$

where U_o is the impingement velocity, r_j the jet radius r the streamwise coordinate measured from stagnation point and p_s the static pressure along a stagnation streamline For application to the present case, we assume $U_o = \Omega l_v \sin\alpha$, $p = p_c$ $r_j = c_v$ and $r = r_v$, so that

$$p_s = p_c + \tfrac{1}{2}\rho(\Omega l_v \sin\alpha)^2[1 - (r_v/c_v)^2] \tag{52}$$

For a fully shielded vertical tail we have

$$C'_{Yv} = \pm\frac{2b_v}{S_w}\left(\frac{\Omega l_v\sin\alpha}{V}\right)^2\int_o^{c_v/2}\left[1 - (r_v/c_v)^2\right] \times \mathrm{d}r_v \qquad \Omega \gtrless 0 \tag{53}$$

or

$$C'_{Yv} = \pm\frac{11S_v}{3S_w}\left(\frac{\omega l_v\sin\alpha}{b}\right)^2 \qquad \Omega \gtrless 0 \tag{54}$$

Under a partially blanketed condition we assume

$$C'_{Yv} = \pm\frac{11A_s}{3S_w}\left(\frac{\omega l_v\sin\alpha}{b}\right)^2 \qquad \Omega \gtrless 0 \tag{55}$$

Also

$$C'_{lv} = C'_{Yv}\left(\frac{h_v}{b}\right) \quad \text{and} \quad C_{nv} = -C'_{Yv}\left(\frac{lv}{b}\right) \tag{56}$$

We observe from Eqs (54) (56) that the magnitudes of C'_{Yv} C'_{lv} and C_{nv} assume significance at combined large angles of attack and spin rates

Input Data

The evaluation of empirical constants embedded in the above theory for the model A airplane (Fig 5) is discussed in the following

Wing

The required empirical constants are C_L C_D and $\bar{x}_{cp}$ In Ref 10 the static wind tunnel test data are presented for body and wing body combinations Ignoring wing body in terference, the wing characteristics are derived by subtracting the body data from the data of wing body combination. The experimental center of pressure data $\bar{x}_{cp}$ is taken from Ref 28

Fuselage

The empirical constants to be evaluated are C_l C_{xc}, and C_{yc} The constant C_l depends on the shape of the fuselage cross section C_{xc} and C_{yc} are aerodynamic parameters and depend on shape, cross flow angle (ϕ), and Reynolds number

The cross sectional shape of fuselage of the model A air plane varies along the length, particularly in the leading edge region (Fig. 6a) However we have to replace it by an ideal fuselage which has a constant cross sectional shape and equal values of S_B dS_B/dx, and vol For this purpose, we refer to Bihrle and Bowman[20] who found that the cross sectional shape of forward part (ahead of center of gravity) does not have much influence on the autorotational characteristics of a fuselage similar to that of the model A airplane In view of this result we assume that the idealized constant cross sectional shape is that which is representative of aft geometry; that is, square section with sharp bottom corners and rounded top (Fig 6a) For this idealized section from the data given by Jorgensen,[23] C_l can be assumed equal to 1 19 For C_{xc} and C_{yc} no comprehensive two dimensional data are available

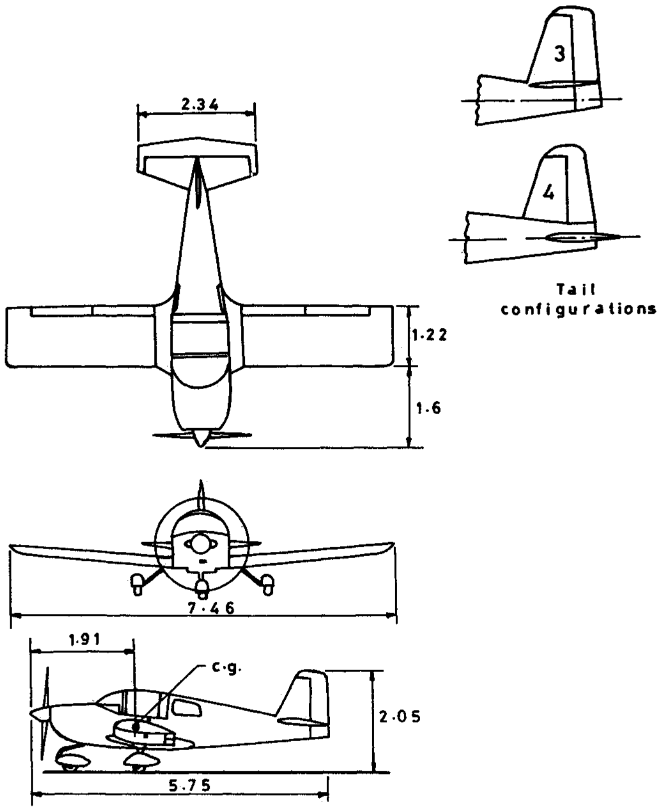


Fig. 5 Spin research airplane—Model A (all dimensions are in meters).

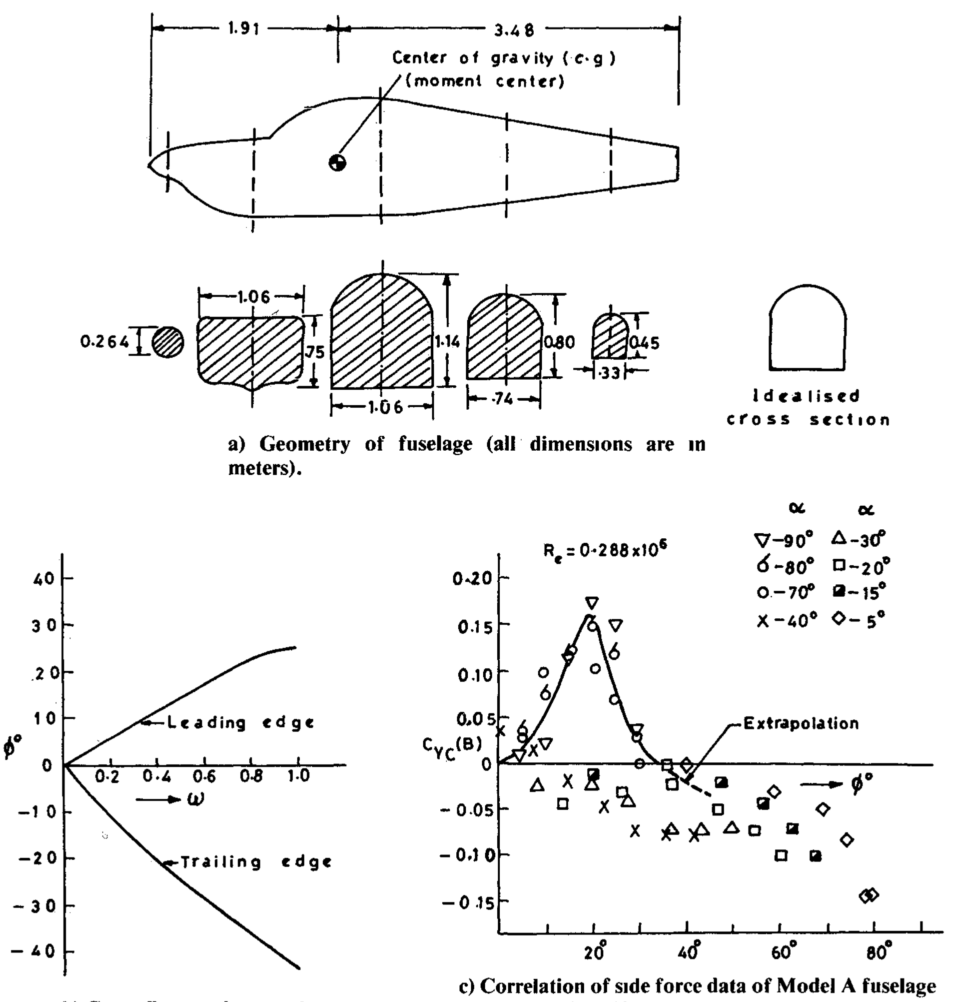
The range of ϕ values [Eq. (27)] encountered in spin are shown in Fig. 6b. The nearest suitable data for C_{xc} is that reported by Polhamus[21] for a square section with 8% corner radius, which is used in this analysis. For C_{yc}, the three dimensional static data[10] of C_{YB} for various α and β can be correlated through Eq. (26) to derive the two-dimensional data on C_{yc}, as shown in Fig. 6c. It may be observed that the data points corresponding to 70 deg $<\alpha<$90 deg give a fair correlation, because at these angles of attack, cross flow is nearly independent of axial flow The scatter in data points for low angle of attack is because of the strong interdependence between axial and cross flows and in such a situation application of Eq. (26) is not valid. Therefore, the only usable part of correlation extends from ϕ=0-30 deg. However, slight extrapolation is necessary to cover the above values of ϕ as shown in Fig. 6c.

Horizontal Tail

The required empirical coefficients C_L, C_D, and x_{cp} are assumed to be identical to those of the wing. It has been assumed that small differences in aspect ratio and airfoil shape do not affect these parameters at high angles of attack.

Vertical Tail

The empirical constant C_{Yv} [Eq. (47)] can be evaluated using Datcom[15] for $\beta_v < B_{v\text{stall}}$. At high angles of attack or side slip, for a vertical tail of low aspect ratio, the variation of C_L or C_Y with α or β, respectively is similar to that of a square plate.[29] With this assumption, Hoerner's[16] data on square plate can be used for vertical tail operating beyond stall.



a) Geometry of fuselage (all dimensions are in meters).

b) Cross-flow angles in right spin.

c) Correlation of side force data of Model A fuselage taken from Ref. 10.

Fig. 6 Input empirical data of fuselage.

The calculated variation of shielded area (A_s) for tails 3 and 4 is shown in Fig 4f

Results and Discussion

The detailed results of this semiempirical theory applied to various other configurations of the model A airplane such as body, wing body, wing-body-horizontal tail etc , will be reported in a NASA publication [30] Here the theory is illustrated with the help of some typical results

For the spin research airplane model A, the free spinning model tests[10] have showed the existence of steady state (equilibrium) spin modes as follows:

Tail No. 3

1) moderately steep spin mode, $\alpha = 50$ deg and $\omega = 0\ 33$

2) flat spin mode, $\alpha = 80$ deg and $\omega = 0\ 62$

Tail No 4

1) steep spin mode, $\alpha = 35$ deg and $\omega = 0\ 218$

2) flat spin mode, $\alpha = 77$ deg and $\omega = 0\ 92$

The rotary balance test data presented in Refs 12 and 13 do not exactly cover all the above angles of attack Therefore, the closest angles of attack where rotary balance data are available are chosen for comparison In Figs 7 10 the predicted rotary aerodynamic coefficients are presented along with the corresponding spin tunnel rotary balance test data [12,13] The incremental coefficients due to rotational flow effect are also marked on these figures In free spinning model tests [10] prospin controls are employed Since the deflection of control surfaces is not considered here the spin tunnel rotary balance test data[12 13] included in Figs 7 10 are also taken for zero control surface deflections This kind of comparison is not a true indication of the actual situation But it is a good representation of the aerodynamic parameters dictating the correspondence spin modes

C_N

At low and moderate angles of attack, wing is the chief contributor to C_N, and at high angles of attack the contribution of the body also becomes significant The rotational flow effect C'_N, increases parabolically with ω for any given α. C'_N is independent of α when both wings are stalled in spin The present theory gives good agreement with rotary balance measurements [12 13]

C_m

Major contribution to C_m comes from the horizontal tail As α increases, the contribution of the body also becomes appreciable The pitching moment is generally negative (stabilizing) at high angles of attack and becomes even more negative with spin rate The agreement between present calculation and rotary balance test data[12 13] is quite satisfactory

C_l

For low and moderate angles of attack, C_l is initially positive (prospin or autorotating) and becomes negative (damping) for higher spin rates Here, wing is the major contributor The quantity C'_l, due mainly to wing, is always damping in nature and increases parabolically with spin rate for $\alpha < 45$ deg As α increases wing contribution diminishes and the rolling moment coefficient comes mainly from

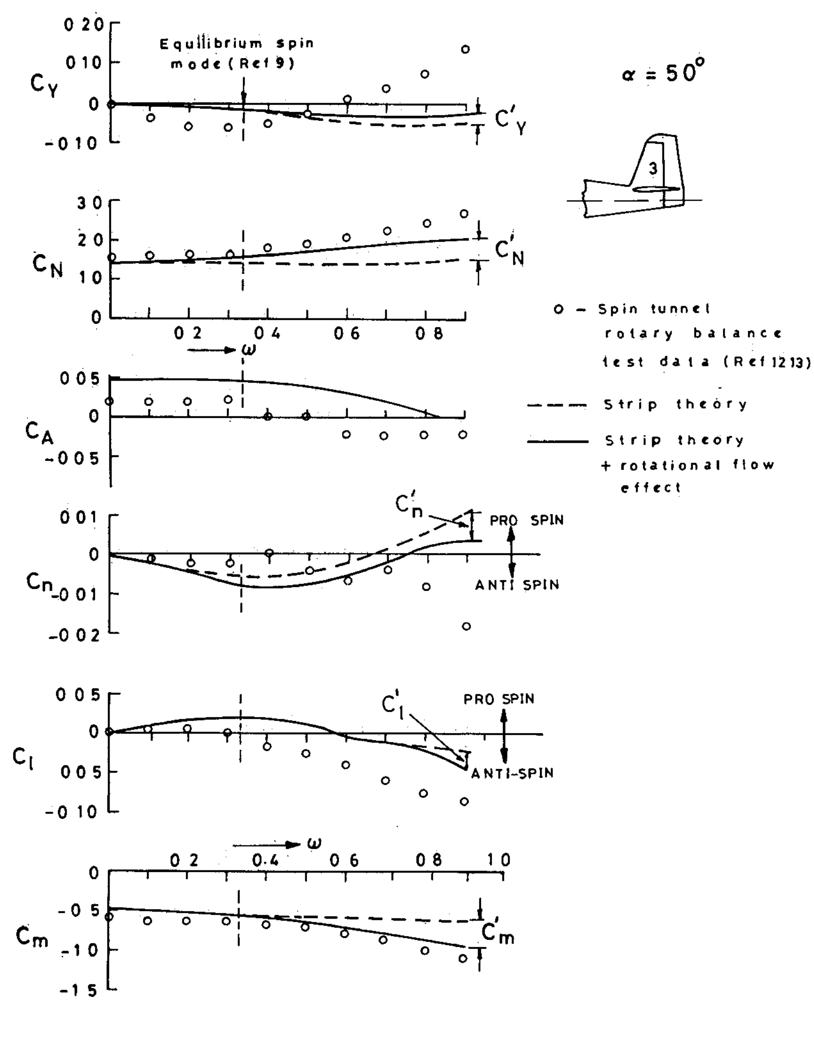


Fig 7 Rotary aerodynamic characteristic of Model A airplane (tail No 3) at moderately flat spin at titude

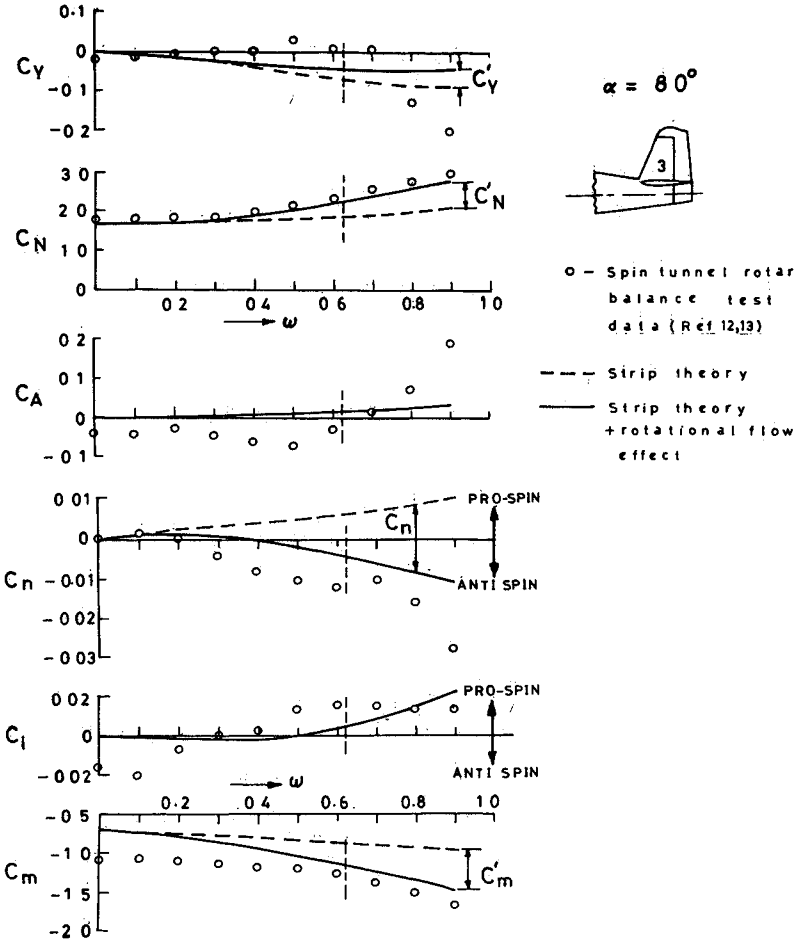


Fig. 8 Rotary aerodynamic characteristic of Model A airplane (tail No 3) at flat spin attitude

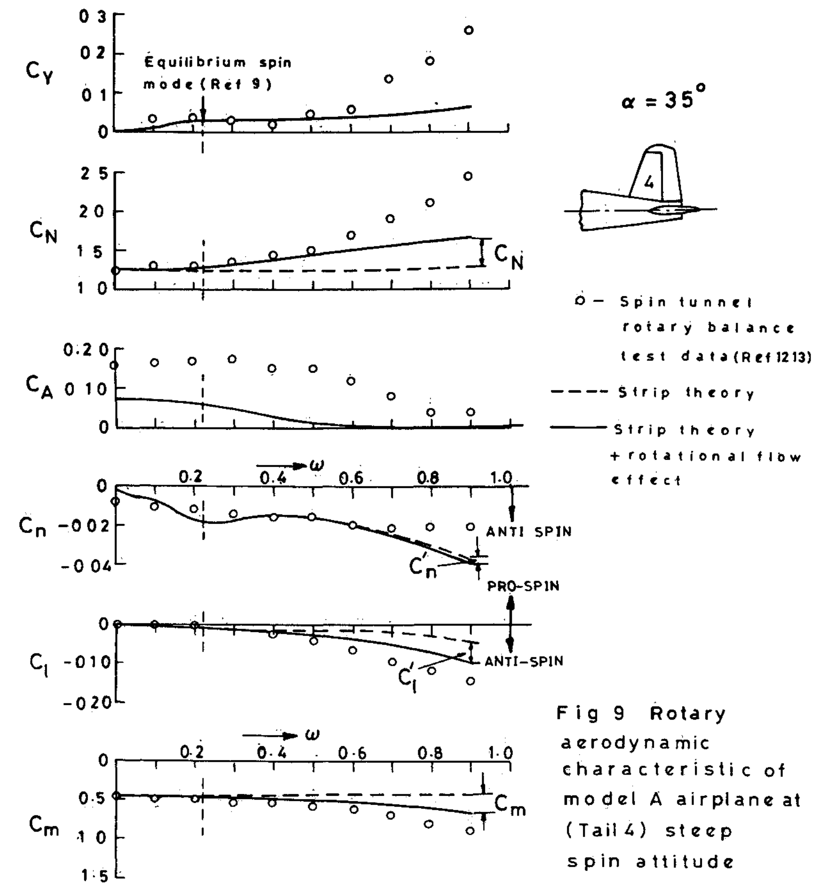


Fig 9 Rotary aerodynamic characteristic of Model A airplane (tail No 4) steep spin attitude

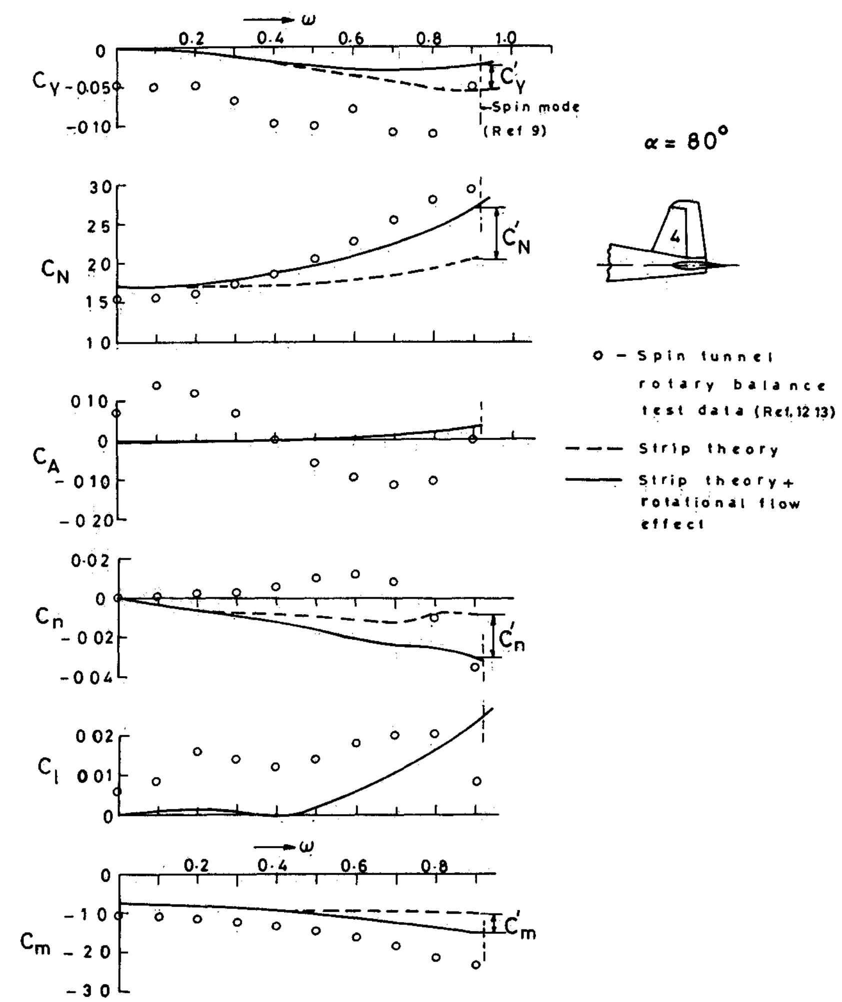


Fig 10 Rotary aerodynamic characteristic of Model A airplane (tail No 4) at flat spin attitude

vertical tail, which is prospin in nature (Figs 8 and 10) The present approach, in general, gives good predictions of C_l However, quantitative differences exist, particularly at high α

C_Y

The fuselage is generally the dominant factor in producing side force in spin The vertical tail's contribution is significant at low α but diminishes as α increases due to shielding effect The present analysis gives fairly good results for steep spin conditions. The disagreement at higher ω for steep spin cases is not of much consequence However for flat spin modes, a noticeable amount of discrepancies exist

C_n

The wing contribution to C_n is generally small The fuselage and vertical tail are the major components producing C_n The contribution of body was generally found to be prospin at all α The vertical tail produces damping moments in yaw, but as α increases, its effectiveness diminishes due to shielding However, interestingly the vertical tail regains its ability to some extent at extreme angles of attack and spin rates (Figs 8 and 10, $\omega > 0\ 5$), which is attributed here to rotational flow effect This trend is consistent with rotary balance measurements [12,13] This fact lends good support to the present rotational flow model for the vertical tail

The predictions of C_n based on present approach are satisfactory for steep spin conditions However, for successful application to flat spin problems, the present theory needs further improvements As discussed in Ref 30, the deficiencies in the present model, particularly for combined high α and ω are possibly because of rotational flow effects for fuselage and fuselage tail interferences which are ignored here

Concluding Remarks

The results of the present aerodynamic mathematical model applied to the spinning motion of a light general aviation airplane are encouraging Some of the important results of this work can be summarized as follows

1) The capability of strip theory, as applied to wing and horizontal tail, has been extended to extreme angles of attack and spin rates by an inclusion of rotational flow effects

2) A beginning has been made to estimate the aerodynamic characteristic of a spinning fuselage having a noncircular cross section A semiempirical method is presented for the prediction of fuselage characteristics at combined high angle of attack and sideslip This method is then extended to the caser of a spinning airplane

3) Mathematical models, based on experimental data, are developed for predicting shielding and rotational flow effects over vertical tail in spin

4) The present semiempirical theory is capable of giving good estimates of aerodynamic coefficients under steep spin conditions However, it needs improvement for successful application to flat spin problems

5) Further studies are necessary in the area of high angle of attack aerodynamics, particularly with regard to noncircular fuselages and understanding the effect of spin rate on such bodies Efforts are required to understand and model the complex interference effects in stall/spin of various aerodynamic surfaces, particularly between body and tail surfaces

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